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Anisotropy Measurements and Cosmological Implications [and Discussion]

R. Fabbri, I. Guidi, V. Natale and R. J. Tayler

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Anisotropy measurements and cosmological implications

By R. Fabbri†, I. Guidi‡ and V. Natale§
† *Istituto di Fisica Superiore, Università di Firenze, Via Panciatichi* 64, *I*-50127 *Firenze, Italy*
‡ *Unità di Ricerca GIFCO/CNR, Università di Firenze, I*-50127 *Firenze, Italy*
§ *Istituto di Ricerca sulle Onde Elettromagnetiche del CNR, I*-50127 *Firenze, Italy*

We summarize the present knowledge of the anisotropies of the cosmic background radiation at angular scales over 1° and present recent data on the dipole and quadrupole harmonics from the Florence group. Reviewing models of cosmic structures, we describe the inferences that can be drawn from the data provided that their origin is extragalactic. We end with a critical discussion of the connection of the background anisotropies with the large-scale properties of the Universe.

1. Introduction

The recent years have provided a substantial advance in our knowledge on the anisotropies of the cosmic background radiation at large angular scales. A first-harmonic (dipole) anisotropy has been unambiguously detected and shown to be independent of frequency (Smoot *et al.* 1977; Cheng *et al.* 1979; Fabbri *et al.* 1980*a*; Weiss 1980). Exciting prospects have been opened by two experiments providing evidence for a more complex structure (quadrupole anisotropy) (Fabbri *et al.* 1980*a*; Boughn *et al.* 1981). While the dipole anisotropy might perhaps be explained by the motion of the solar system inside the Virgo supercluster, the few higher-order harmonics, if they are not simulated by emissions from our galaxy, are connected with the structure of the Universe on very large scales. Therefore a definitive demonstration of the extragalactic origin of the detected quadrupole is an important goal to be reached in the next few years. At the same time, theoretical investigations have carried a deeper insight in the possible relations between the background anisotropies and the cosmic structure.

In this paper we first summarize the experimental knowledge of the background anisotropies at angular scales larger than 1°. In particular, we present preliminary results on the first-order and second-order harmonics, which we derived from a more complete set of data collected during the 1978 Italian experiment. We also discuss the tentative evidence for anisotropies at the angular scale of 6°. In §3 we treat the cosmological inferences that one can draw from the experiments, provided that the signals are extragalactic. We hope that not many important omissions may be found in our attempt to cover the range of theoretical models. In §4 we shall adopt a more general point of view, and discuss how fairly direct information on the cosmic structure can be derived with a minimal number of theoretical assumptions. Fabbri *et al.* (1981) pointed out that the anisotropy of the background radiation can provide a sort of map of the Universe. However, we should not forget that we can construct only a two-dimensional map at best, and inferences about the three-dimensional Universe require some extrapolations that are not trivial when the lowest-order harmonics are considered. Our discussion may be considered as an attempt to clarify the meaning of the usual assumptions.

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2. Observational data

The approach to the anisotropy of the background radiation can be either deterministic or stochastic. In the former case one looks for $T(\theta, \phi)$, the distribution of the radiation temperature over the celestial sphere (here θ and ϕ are arbitrary polar coordinates). In practice it is convenient to measure the harmonic content of the anisotropy. If we set

$$T(\theta, \phi) = T_0 \sum_{l,m} a_{lm} Y_{lm}(\theta, \phi), \tag{1}$$

Table 1. Recent dipole and quadrupole measurements

	Berkeley	Princeton	Florence
T_x/mK	-2.3 ± 0.7	-3.68 ± 0.19	$(-2.43 \pm 0.25)\, C$‡
T_y/mK	0.4 ± 0.7	0.43 ± 0.19	$(0.65 \pm 0.15)\, C$
T_z/mK	63 ± 29	-0.39 ± 0.16†	$(-1.97 \pm 0.20)\, C$
Q_1/mK	40 ± 19	—	—
Q_2/mK	-1.1 ± 0.6	0.28 ± 0.22	$(0.10 \pm 0.12)\, C$
Q_3/mK	0.0 ± 0.5	0.13 ± 0.21	$(0.50 \pm 0.09)\, C$
Q_4/mK	0.1 ± 0.3	-0.31 ± 0.15	$(-0.30 \pm 0.11)\, C$
Q_5/mK	-0.1 ± 0.4	-0.54 ± 0.14	$(-0.13 \pm 0.11)\, C$

† The fitted parameter is $T_z + 1.12 Q_1$.
‡ $C = 1.0^{+0.4}_{-0.2}$ is the calibration factor.

where $Y_{lm}(\theta, \phi)$ is the spherical harmonic, the three terms with $l = 1$ describe the dipole, the five terms with $l = 2$ the quadrupole, etc. The goal is then to measure the coefficients a_{lm}. This has so far been done only for the dipole and quadrupole harmonics.

The second approach is to investigate the r.m.s. fluctuations of the angular distribution and is the only available one at fine scales. In a double-beam experiment one measures the temperature difference between two spots in the sky separated by an angular distance β, and derives the r.m.s. value

$$\langle [\Delta T(\beta)]^2 \rangle = 2[\Gamma(0) - \Gamma(\beta)], \tag{2}$$

where $\Gamma(\beta)$ is the correlation function of the cosmic background. The representations (1) and (2) are related to each other. If the r.m.s. anisotropy at a scale β is originated by harmonics with $l \gtrsim \pi/\beta$, then

$$\frac{\Delta T}{T} \approx \frac{1}{4\pi} \sum_{l,m} |a_{lm}|^2. \tag{3}$$

If on the other hand it is produced by low-l harmonics,

$$\frac{\Delta T}{T} \approx \frac{\beta^2}{8\pi} \sum_{l,m} l(l+1)\,|a_{lm}|^2. \tag{4}$$

Tables 1–3 summarize recent results following the first and second approach, at angular scales greater than 1°. We only select up-to-date experiments, which best describe the present knowledge of the background anisotropies. The data on the spherical harmonic expansion in table 1 are expressed in the Berkeley notation, where the angles ϕ and $\frac{1}{2}\pi - \theta$ are identified with right ascension α and declination δ respectively, and

$$-T_x + \mathrm{i}T_y = (3/2\pi)^{\frac{1}{2}} a_{11}; \quad T_z = (3/4\pi)^{\frac{1}{2}} a_{10};$$
$$Q_1 = (5/4\pi)^{\frac{1}{2}} a_{20};$$
$$-Q_2 + \mathrm{i}Q_3 = (15/8\pi)^{\frac{1}{2}} a_{21}; \quad Q_4 - \mathrm{i}Q_5 = (15/8\pi)^{\frac{1}{2}} a_{22}.$$

The Berkeley data are taken from Gorenstein & Smoot (1981) and refer to observations at a wavelength of 0.89 cm. The Princeton data are derived from measurements at four wavelengths between 1.5 cm and 6.5 mm (Boughn *et al.* 1981). The Florence data refer to the 0.5–3 mm band.

The dipole anisotropy appears to be independent of wavelength, as we expect for a Doppler effect. This is more remarkable if we consider that completely different equipment is used in

TABLE 2. CORRELATION MATRIX OF 1978 FLORENCE EXPERIMENT

	T_0	T_x	T_y	T_z	Q_2	Q_3	Q_4	Q_5
T_0	1	−0.15	0.34	−0.04	−0.08	−0.07	−0.21	−0.07
T_x		1	−0.43	−0.80	−0.90	0.30	−0.85	−0.40
T_y			1	0.43	0.33	−0.83	−0.25	0.90
T_z				1	0.80	−0.46	−0.48	0.46
Q_2					1	−0.29	−0.81	0.38
Q_3						1	0.09	−0.90
Q_4							1	−0.23
Q_5								1

TABLE 3. INTERMEDIATE ANGULAR SCALE RESULTS

reference	wavelength/mm	β/deg	ΔT/mK
Fabbri *et al.* (1980*b*)	0.5–2	2	$\leqslant 0.9$
Mandolesi *et al.* (1981)	28	2.3	$\leqslant 0.6$
Fabbri *et al.* (1979) } Melchiorri *et al.* (1981) }	0.5–2	6	$0.11^{+0.04}_{-0.02}$
Gorenstein & Smoot (1981)	8.9	7	$\leqslant 0.5$
this work	0.5–2	6	$0.25^{+0.10}_{-0.05}$

the radio and infrared regions. The situation is less clear for the quadrupole anisotropy. The radio data are not likely to suffer from Galactic emission, but the statistical significance of the quadrupole fitting is not as high as we should like. The Florence data give better evidence for a distortion of the dipole pattern, but the measured quadrupole may be affected by Galactic dust.

The original paper by Fabbri *et al.* (1980*a*) did not exhibit a fitting with the various quadrupole components. Only data collected in the first part of the 1978 flight were used for that analysis. The third column of table 1 reports the up-to-date results of a more complete analysis, taking into account data collected during the entire flight. It is worth describing such (so far unpublished) results in some detail. Although 4 h of data are employed scattered over an interval of 9 h, the limited sky coverage does not yet allow the simultaneous fitting of all of the second-order harmonics. Thus Q_1, which was strongly correlated with T_z, has not been included in the fitting. Even leaving Q_1 aside. the correlation coefficients of the combined eight-parameter fitting are still quite large, as shown in table 2.

The errors shown in the third column of table 1 are statistical only (at 1 standard deviation). Thus a calibration factor C is included, taking into account the calibration errors

$$C = 1.0^{+0.4}_{-0.2}.$$

The fitting gives $\chi^2 = 166$ for 160 degrees of freedom when the variance is set equal to sky fluctuations on a scale of 6°(0.1–0.25 mK) rather than to the detector noise (0.06 mK; see below). There seems to be a fair agreement with the Princeton results on the whole. There is a very close agreement on Q_4, but this may be accidental for the discrepancy on the T_z component of the dipole.

The problem of possible spurious contributions to the quadrupole anisotropy is in our opinion interlaced with the question of the background anisotropies at lower scales. Thus we should consider the data available at $\beta > 1°$.

Table 3 shows that upper limits of $\Delta T/T$ on the order of 10^{-4} have been given at scales of *ca.* 2°. But, more interestingly, a positive detection has been reported at $\beta = 6°$. Fabbri *et al.* (1979) observed random fluctuations of 0.11 mK at Galactic latitudes $b \geqslant 20°$, and suggested that they might have a cosmological origin. Melchiorri *et al.* (1981) have analysed the correlation of the detected signals with a dust indicator (the neutral hydrogen column density); their conclusion is that Galactic dust emission is perhaps insufficient to account for such fluctuations, and its large-scale gradient should be about one-fifteenth of the quadrupole gradient. This conclusion may be weakened by possible variations of the dust:gas ratio at large values of b. In the sky regions observed in the second part of the 1978 flight larger fluctuations have been detected, with an r.m.s. value of 0.25 mK. The detector noise is 0.06 mK for an average observation time of 10^2 s for each sky beam. The sky fluctuations emerge quite clearly and show a sawtooth structure. Another interesting question is whether there can exist an appreciable contribution of extragalactic origin. Melchiorri (1982) finds evidence for signals from the Virgo cluster and attempts an interpretation in terms of the Sunyaev–Zel'dovich effect. However, this would be a well localized signal, having no strong effect on the detected quadrupole and the r.m.s. fluctuation at $\beta = 6°$.

Clearly, we need more data for a definite conclusion. A second experiment has been flown by Melchiorri & Natale in 1980, and a third one by Natale in 1981. In particular, the latter explored three frequency ranges around 1 mm, 800 μm and 500 μm. The submillimetric channels are intended to monitor the Galactic dust emission, and a comparison of the detected signals should allow one to distinguish the anisotropies intrinsic to the background radiation from spurious emissions. The analysis of the data collected in these two flights is in the course of execution. Also, another balloon flight is planned for 1983. Smoot, Lubin & Epstein recently had a new balloon flight with a 3 mm radiometer and a second one is in preparation (G. F. Smoot, personal communication). An accuracy of 0.1 or 0.2 mK is anticipated. Wilkinson's group is analysing the data from their three maser flights (D. T. Wilkinson, personal communication). It seems to us that, when a sufficient sky coverage is obtained, the sensitivity of the current experiments should allow an unambiguous interpretation of the second-order anisotropy, and possibly the detection of higher-order harmonics.

3. The origin of the large-scale anisotropy

From the discussion of §2 we can conclude that the centimetric and millimetric backgrounds are really anisotropic, although we cannot yet be sure that the dominant cosmic component has an angular pattern more complex than dipolar. As to the origin of the dipole anisotropy, we can safely state that it is basically due to Doppler shift. (However, if the motion involves a large portion of the Universe any distinction between Doppler and gravitational effects is meaningless.) In fact the velocity field within the Virgo supercluster is expected to produce a dipole anisotropy of the correct order of magnitude. However, the red-shift field of nearby galaxies implies a motion of the Solar System inconsistent with the microwave data. Although the optical analyses are not very consistent with each other (see, for example, de Vaucouleurs *et al.* 1981; Tammann *et al.* 1979), our motion in the supercluster frame seems to be roughly at right angles to the dipole direction. One might regard this discrepancy as evidence for cosmic structure on scales $L \gtrsim 10^2$ Mpc. At any rate, considering the large-scale properties of our Universe is mandatory if the quadrupole anisotropy is genuinely extragalactic.

Let us therefore consider cosmological models where a quadrupole anisotropy is predicted in a natural way. Most of them are linearized models, where small perturbations are superimposed on the homogeneous and isotropic background provided by the Friedmann cosmology. Perturbative modes were first considered in the general relativistic context by Lifshitz & Khalatnikov (1963) and classified as density, velocity and gravitational waves. More refined treatments of density perturbations have been employed in the cosmological theories of galaxy formation, where a proper distinction is made between isothermal and adiabatic waves (for a review see Jones 1976). Also, some exact solutions are available, where a large-scale density variation is treated by nonlinear techniques. These are idealized solutions, endowed with special symmetries. Finally, we should consider homogeneous anisotropic models. Here the Universe is assumed to be spatially homogeneous, but the Hubble expansion is anisotropic for any observer. A cosmic velocity field is often included (for a review see MacCallum 1979). These models are useful because they give an approximate description of inhomogeneous perturbations when their length scale is much larger than the particle horizon.

In models with density fluctuations, anisotropies of the cosmic background are produced by the intrinsic variations of the radiation density at different points of space (Sunyaev & Zel'dovich 1970) as well as by Doppler and gravitational frequency shifts (Sachs & Wolfe 1967). In the remaining models, of course, only the latter mechanisms are present (Collins & Hawking 1973).

Homogeneous models, although endowed with a limited number of degrees of freedom, offer a fairly wide range of anisotropy patterns. The angular variations are generated mainly by the quadrupole or by higher-order harmonics (see, for example, Collins & Hawking 1973). The dipole term turns out to be much smaller than the quadrupole except for models that we do not expect to provide anisotropies as large as *ca.* 10^{-3} (Fabbri 1980). Homogeneous models are therefore good candidates for explaining the quadrupole anisotropy but not the dipole.

Inhomogeneous models can be adjusted more easily to any given harmonic pattern. By analogy with experiment, we find two basic types of approach to the problem. Some of the current models try to fit the available data in detail by means of some large-scale inhomogeneity. Other models are statistical because they consider stochastic distributions of inhomogeneities that are intended to provide expectation values for the harmonic amplitudes or r.m.s. fluctuations consistent with experiment.

Models of the first type consider simple distributions of matter: Raine & Thomas (1981) assume a spherically symmetric bump centred at red shift $z \approx 2$, whereas Fabbri *et al.* (1982) choose a monochromatic plane wave. It is remarkable that a rough agreement with the available data can be obtained when the linear scale of the perturbation λ becomes comparable with the particle horizon or the size of the visible Universe. More precisely, the first-order and second-order harmonics have comparable magnitudes when $\lambda \approx c/H_0$, where H_0 is the Hubble constant; the quadrupole prevails at still larger scales, whereas the dipole becomes somewhat larger for $\lambda \approx 10^3$ Mpc. Of course we should not demand too much from highly idealized models. Spherically symmetric configurations, although they are common for relaxed systems, are unlikely to occur for small density contrasts ($\Delta\rho/\rho \approx 10^{-2}$ in the model of Raine & Thomas). Deviations from spherical symmetry would change the harmonic content of the anisotropy, so that the difficulty later pointed out by Raine & Thomas (1982) is not conclusive against single-lump models. Likewise a monochromatic plane wave should not be expected to explain all the details of the data. We should be satisfied with the approximate agreement, and with the prediction that waves with $\lambda \approx c/H_0$ should originate several other harmonics possibly detectable in the near future (Fabbri *et al.* 1982).

Velocity and gravitational waves, too, have been considered in the literature for both small and very large wavelengths. A quadrupole pattern arises from them for $\lambda \gg c/H_0$ (Grishchuck & Zel'dovich 1978).

Current models following the statistical approach present a distinguishing feature, i.e. they introduce a continuous spectrum of perturbations with no preferred scale. The indeterminacy arising from this approach is remedied by considering the current theories of galaxy formation or the observable properties of the galaxy distribution. Thus Silk & Wilson (1981) have calculated the expectation values of the lowest few multipoles and the r.m.s. fluctuation arising from a random system of plane waves assuming power laws for the primordial spectrum of the perturbations. Namely, if the Fourier component of the density contrast $\Delta\rho/\rho$ is $\delta(\boldsymbol{k})$, they set

$$|\delta(\boldsymbol{k})|^2 \propto k^n. \tag{5}$$

They find that, if the quadrupole is fitted to the observed value, then the dipole can be explained only by isothermal perturbations with $n \approx 0$. However, the situation becomes much more favourable also for adiabatic perturbations if the standard scenario of galaxy formation is modified by the introduction of a cosmic background of massive neutrinos (Silk & Wilson 1981; Silk 1982).

Peebles (1981, 1982*a*, *b*) considers a random distribution of spherically symmetric condensations. The large-scale fluctuations in the distribution of clumps (shot noise: $|\delta(\boldsymbol{k})| = \text{constant}$) give rise to low-order harmonics in the angular pattern of the cosmic background. This attractive theory has the advantage of presenting no free parameters (except the cosmic density parameter, Ω_0, which is a free parameter in any theory), and leads to the correct order of magnitude for both the quadrupole anisotropy and the reported fluctuation at $\beta = 6°$. It does not predict a sufficiently large dipole anisotropy. Models like Peebles's could be implemented by local supercluster models such as that due to White & Silk (1979), provided that the discrepancy between microwave and optical data is resolved.

4. The background anisotropy as a picture of the Universe

In this section we discuss how direct information on the cosmic structure can be derived from measurements of the background anisotropies. Leaving aside the problem of Galactic emission, it still turns out that a given anisotropy pattern can be produced by different distributions of matter, and a choice can be made only by means of assumptions that should be stated clearly. We cannot be sure either that the anisotropy is generated entirely by density fluctuations, since vorticity or gravitational waves would produce similar effects. However, in the following we shall focus on density waves because matter inhomogeneities are directly observed at low scales. (Homogeneous models will be included implicitly as a limiting case.) We then ask, to what extent the anisotropy pattern determines a density wave pattern (Fabbri *et al.* 1982)? Express the wave system as a superposition of multipoles

$$\Delta\rho/\rho = \sum_{k,l,m} C_{klm} f_{kl} Y_{lm}(\theta, \phi), \tag{6}$$

with f_{kl} depending on the radial coordinate and time (Lifshitz & Khalatnikov 1963). For wavelengths larger than the optical horizon at the last scattering epoch the coefficients of the expansion (1) are directly related to C_{klm} by

$$a_{lm} = \sum_k C_{klm} I_{kl}. \tag{7}$$

Each wave multipole generates a correspondent spherical harmonic in the background anisotropy, but all the wavenumbers k contribute. The coefficients I_{kl} have been calculated by Fabbri *et al.* (1982) for the perturbative modes that grow up with time.

Measurements of the harmonic amplitudes a_{lm} are equivalent to a two-dimensional photograph of the Universe, so the spectrum of k cannot be determined from the comparative analysis of a_{lm} without some further assumption. We expect perturbations with wavelength $\lambda = 2\pi/k$ to give the largest fluctuations at the angular scale $\beta \approx c\lambda H_0$, in addition to a dipole term arising from Doppler shift at the observer site. In a flat space, for a given k the function I_{kl} exhibits a sharp cutoff for $l > k/cH_0$ and oscillates with a small variation of amplitude for $2 \leqslant l \ll k/cH_0$. Moreover, wave systems with a fixed k usually present either a smooth dependence of C_{klm} or a cutoff suppressing the multipoles with large l. Since the number of spherical harmonics in the range $(\frac{1}{2}l, l)$ is *ca.* l^2 for random distribution of inhomogeneities (and *ca.* l for cylindrically symmetric patterns) the preferred angular scale β_0 of the anisotropies is determined by the largest l that is not suppressed, namely $\beta_0 \approx \pi/l_{\max} \approx k/cH_0$.

However, the reverse proposition is not true, since the largest contribution to a_{lm} may easily come from larger k since the number of available modes increases as k^3 (as is apparent when we consider plane wave modes). In the theories of Silk & Wilson (1981) and Peebles (1982*a*, *b*), considering the spectra of (5), this circumstance occurs for $n \geqslant 0$. This does *not* imply that the quadrupole anisotropy is produced directly by some nearby, small-scale clump in such models. A given wavenumber k produces inhomogeneities mainly on a scale $2\pi/k$, but also some fluctuations on much larger scales. A random superposition of short waves or spherically symmetric clumps can provide some excess of matter in large regions of space and consequently a quadrupole anisotropy in the background radiation.

Therefore the first question to be answered is, whether the quadrupole originates from the fluctuation in the distribution of galaxies or from a fairly simple structure emerging at some large scale. Note that the latter scenario should arise also from a power law spectrum with a sufficiently negative n: if a scale of the order of the particle horizon dominates, only one or few clumps of that size can exist in the visible Universe. Thus the contraposition of statistical and deterministic models is weakened.

The possible discovery of a cosmic structure of the size of the visible Universe or larger is an exciting prospect, but we should face the unpleasant fact that $\Delta\rho/\rho$ is not measured directly. A valuable help would be provided by the detection of large-scale anisotropies in another cosmic background, emitted at red shifts appreciably smaller than the last scattering red shift of the microwave background. Fabian *et al.* (1980) have claimed the existence of a residual anisotropy of the X-ray background after the Galactic contribution has been subtracted. Unfortunately the Galactic contamination is large and no firm conclusions can be drawn at present. However, if we assume that the effect is extragalactic, it is possible to fit it into a model explaining the anisotropy of the microwave background (Raine & Thomas 1981). More generally, we observe that I_{kl} depends on the emission red shift. For moderate values of k and $\Omega_0 = 1$,

$$I_{kl} \approx \tfrac{1}{15}kR\,\delta_{l1} - \tfrac{1}{5}j_l(kR), \tag{8}$$

where j_l is a spherical Bessel function and R the difference in size between the optical horizons at the observation and emission epoch. Comparing the harmonic amplitudes of a large and a small red-shift background in principle we can determine k, the wavenumber that remains unknown if we limit ourselves to a single photograph of the Universe. Although in practice we should consider an unavoidable spreading of the k spectrum, there is still the possibility of determining the effective k dominating each spherical harmonic.

The task of determining the large-scale structure of the Universe is very difficult, although the cosmic background is a sensitive probe. The discovery of filamentary systems and big holes on scales of the order of 10^2 Mpc (Einasto *et al.* 1980; Kirshner *et al.* 1981) may suggest the existence of similar complexities on the scale of the particle horizon or more. We may have to face awkward situations, where we can take advantage of neither symmetry and simplicity nor full disorder.

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Note added in proof (13 *July* 1982). According to a recent experiment (L. Hart & R. D. Davis, *Nature, Lond.* **297**, 191–193 (1982)), the motion of the Solar System with respect to nearby galaxies would agree with the cosmic background dipole data.

A preliminary analysis of the 1981 Florence multichannel flight shows some correlations between millimetre and submillimetre fluctuations at an angular scale of 6°, contrasting with the arguments of Ceccarelli *et al.* (1981).

References

Boughn, S. P., Cheng, E. S. & Wilkinson, D. T. 1981 *Astrophys. J. Lett.* **243**, L113–L117.
Cheng, E. S., Saulson, P. K., Wilkinson, D. T. & Corey, B. E. 1979 *Astrophys. J. Lett.* **232**, L139–L143.
Collins, C. B. & Hawking, S. W. 1973 *Mon. Not. R. astr. Soc.* **162**, 307–320.
de Vaucouleurs, G., Peters, W. L., Bottinelli, L., Gauguenheim, L. & Paturel, G. 1981 *Astrophys. J.* **248**, 408–422.
Einasto, J., Joever, M. & Saar, E. 1980 Superclusters and galaxy formation. *Nature, Lond.* **283**, 47–48.
Fabian, A. C., Warwick, R. S. & Pye, J. P. 1980 *Physica Scr.* **21**, 650–651.
Fabbri, R. 1980 *Phys. Lett.* A **79**, 21–22.
Fabbri, R., Guidi, I., Melchiorri, F. & Natale, V. 1979 Presented at 2nd Marcel Grossmann Meeting, Trieste.
Fabbri, R., Guidi, I., Melchiorri, F. & Natale, V. 1980*a* *Phys. Rev. Lett.* **44**, 1563–1566.
Fabbri, R., Melchiorri, B., Melchiorri, F., Natale, V., Caderni, N. & Shivanandan, K. 1980*b* *Phys. Rev.* D**21**, 2095–2102.
Fabbri, R., Melchiorri, B. & Melchiorri, F. 1981 *Adv. Space Res.* **1**, 19–32.
Fabbri, R., Guidi, I. & Natale, V. 1982 *Astrophys. J.* **257**. (In the press.)
Gorenstein, M. V. & Smoot, G. F. 1981 Large-angular-scale anisotropy in the cosmic background radiation. *Astrophys. J.* **244**, 361–381.
Grishchuck, L. P. & Zel'dovich, Ya. B. 1978 *Soviet Astr.* **22**, 125–128.
Jones, B. 1976 *Rev. mod. Phys.* **48**, 107–149.
Kirshner, R. P., Oemler, A., Schechter, P. L. & Schectman, S. A. 1981 *Astrophys. J. Lett.* **248**, L57–L60.
Liftshitz, E. M. & Khalatnikov, I. M. 1963 *Adv. Phys.* **12**, 185–249.
Mandolesi, N., Morigi, G., Inzani, P. & Sironi, G. 1981 *Boll. SIF* **124**, 18.
MacCallum, M. A. H. 1979 In *General relativity, an Einstein centenary survey* (ed. S. W. Hawking & W. Israel), pp. 533–580. Cambridge University Press.
Melchiorri, F., Melchiorri, B., Ceccarelli, C. & Pietranera, L. 1981 *Astrophys. J. Lett.* **250**, L1–L4.
Melchiorri, F. 1982 Preprint, University of Rome.
Peebles, P. J. E. 1981 *Astrophys. J. Lett.* **243**, L119–L132.
Peebles, P. J. E. 1982*a* In *Cosmology and fundamental physics.* (*Ser. varia pont. Acad. Sci.* **48**). (In the press.)
Peebles, P. J. E. 1982*b* *Astrophys. J.* (In the press.)
Raine, D. J. & Thomas, E. G. 1981 *Mon. Not. R. astr. Soc.* **195**, 649–660.
Raine, D. J. & Thomas, E. G. 1982 (In preparation.)
Sachs, R. K. & Wolfe, A. M. 1967 *Astrophys. J.* **147**, 73–90.
Silk, J. 1982 In *Cosmology and fundamental physics* (*Scr. varia pont. Acad. Sci.* **48**). (In the press.)

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Silk, J. & Wilson, M. L. 1981 *Astrophys. J. Lett.* **244**, L37–L41.
Smoot, G. F., Gorenstein, M. V. & Muller, R. A. 1977 *Phys. Rev. Lett.* **39**, 898–901.
Sunyaev, R. A. & Zel'dovich, Ya. B. 1970 *Astrophys. Space Sci.* **7**, 3–19.
Tammann, G. A., Yahil, A. & Sandage, A. 1979 *Astrophys. J.* **234**, 775–784.
Weiss, R. 1980 *A. Rev. Astr. Astrophys.* **18**, 489–535.
White, S. & Silk, J. 1979 *Astrophys. J.* **231**, 1–9.

Discussion

R. J. TAYLER (*Astronomy Centre, University of Sussex, U.K.*). I agree with Dr Fabbri's comment that it is difficult to deduce three-dimensional structure from two-dimensional observations. However, some progress can presumably be made by assuming that the characteristic scale in the radial dimension is comparable with the scale perpendicular to the line of sight.

R. FABBRI. Professor Tayler is quite right as far as small-scale anisotropies are considered. However, any inferences become more uncertain in connection with the lowest order harmonics. In particular, unless all the data at intermediate and large angular scales are readily interpreted by some simple theory (which is difficult if ΔT is really a decreasing function of β around 6°), it may be hard to decide which wavenumber range should be linked with the quadrupole anisotropy.

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